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Abstract

In this paper we analyse a common value English auction. We argue that rational bidders attempt to estimate each other's private signals, to take advantage of the additional information disclosed through the bids. If this happens, herd behaviour might arise, because a particular bidder may have an incentive to follow his estimate of some other bidder's signal, thus dropping his own, and staying in the auction longer than previously optimal. Acting upon beliefs might take the auction to an inefficient outcome, where the bidder who most values the good ends up not getting the object for sale.

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1 Introduction

English or ascending auctions are probably the most common type of auction, as suggested for example by McAfee and McMillan [4], and are surely the easiest to organise and conduct. In the auction literature, especially in Milgrom and Weber [5], it is suggested that it maximizes revenue for the seller when compared with other auction formats. The main difference between an English auction and the sealed bid (first-price or second-price) and Dutch auctions is the bidding process. In the former, several increasing bids are submitted by each bidder whereas in the latter only one is submitted. In a common value setting, where the good for sale has an unknown but common value for every bidder, knowing the drop out prices of other bidders reveals information. Knowing the bidding function and the drop out price, one can infer the initial signal received by that particular bidder, and this lessens the winner's curse. Hence, the remaining bidders will be less afraid of bidding more aggressively, which increases the final price and consequently the expected revenue for the seller, when compared to the expected revenue of the other auction formats.

However, this and other models of English auctions seem to have disregarded the behavioural aspects involved with the bidding process. An example of this is the strategic equivalence assumed to exist between English and second-price sealed bid auctions when only two bidders are involved. We argue that in the two bidder case, the strategies and outcomes of an English auction could be quite different from the ones in a second-price sealed bid auction. In a common value framework, the increasing bids are sequentially revealed, and this may cause the two bidders to revise their strategies in a manner similar to the drop out prices in Milgrom and Weber. When a bidder submits a bid, this must surely come from his bidding strategy, which naturally depends on his information set. But if this is the case, then his opponent may use the current bid to try and infer his information set. Remember that in a common value model, the value of good for sale is common but unknown, so each bidder's estimate of the unknown value is valuable information. In our model, each bid is considered to be a temporary drop out price, which becomes the effective price if one of the bidders quits.

This inference of the opponent's information set is done through a Bayesian updating process, which takes into account the current bid as carrying important information and which may then cause that bidder to change his bidding rule. The updating process will give this bidder a more or less accurate estimate of the other bidder's initial signal, which he may than use to try and take advantage of that additional information. If he does make use of this information, we will say the he is a "rational bidder". In this case, and in a manner similar to Milgrom and Weber [5], the final price will be higher relative to the outcome of a second-price sealed bid auction, even in the case of two bidders. And the expected revenue for the seller will also be higher.

This behavioural aspect, a consequence of the English auction design, may have other effects. Once the Bayesian updating process is triggered, herd behaviour could occur. Banerjee [2] or Scharfstein and Stein[6] provide examples where it totally rational to drop one's private information and follow someone else who we believe has better information than we do. In a common value framework, this could well be the case. Bidders will have estimates of the unknown value of the good, but some could contain more information than others. Because the Bayesian updating process provides estimates on our opponent's initial signal, we will then be able to compare it to our initial signal and decide whether dropping our initial signal is profitable or not. If herd behaviour does occur, it will only deviate the outcome of an English auction even further from the second-price sealed bid auction.

A side result of our model suggests that when the Bayesian updating process is triggered, not only will the final price be relatively higher but it is also possible that the bidder who received the highest initial signal ends up not winning the auction. If this is the case, the outcome will be inefficient. The argument for this result is that the low signal bidder may overestimate his opponent's signal (through the Bayesian updating process), and stay active longer than he should. If he stays too long, he will drive the high signal bidder out, win the auction and almost surely realize a negative payoff. This result is comparable to real world auctions, where it is often the case that the winner ends up with a negative profit.

In an experimental study, Avery and Kagel [1] also found evidence that a significant proportion of auctions (30% for experienced bidders and 39% for inexperienced bidders) cause the winner to receive negative profits. This naturally contradicts the Nash symmetric equilibrium prediction that not only the winner will make positive profits but also that the winner is the high signal bidder. Our model, through the Bayesian updating process, addresses this evidence. Bidders, by inverting the bids in an attempt to estimate their opponent's signal, revise their bidding rules, and will almost surely be willing to stay active longer than predicted by the Nash equilibrium strategies. In an English auction, staying active longer means that the final price is necessarily higher, and this could explain why the winner receives negative profits. If the low signal bidder stays active too long (i.e. when they overestimate their opponent's signal), he may even win the auction, and realize substantial negative profits.

The paper is structured in the following way: the next Section presents the model in the presence of rationality; Section three introduces the possibility of herd behaviour; and Section four concludes.

2 The Basic Model with Rationality

Our model will be based on the English auctions' section in Milgrom and Weber [5]. Like them, we are dealing with a second-price auction, in which when there is only one bidder left (who would have just placed his winning bid), this bidder is the winner and will pay the penultimate's bidder final bid. For notational purposes, capital letters denote random variables, lower case letters denote their realizations, capital letters in bold denote matrixes and a capital letter with a hat denotes an estimate, for example, an expected value. We are assuming one seller, one good for sale and 2 bidders, or potential buyers. Every bidder is given an initial private signal about the value of the good. Let us denote this signal by X_i, with $i = 1, 2$, and let us denote $X = (X_i, X_{-i})$ for $i = 1, 2$ as the informational vector. We will hence forth denote as bidder $-i$ the other bidder, when we are speaking about bidder i; in a clear abuse of notation. The value of the good is *ex post* the same for both bidders, but it is *ex ante* unknown, and given by the random variable S. This is a common assumption for a common value auction. We should also note two other things: S may never be known and the realization of X_i, x_i, is known only to bidder i.

Let us then assume that bidders bid alternately, and that each bidder's valuation function depends on the vector of initial signals and on S. Milgrom and Weber denote the valuation function as $V_i = v_i(X_i, X_{-i}, S) = v_i(X, S)$, which would make bidder i's expected valuation dependent *ex ante* only on his own private signal, since this is the only argument in $V_i(.)$ known to him. We then assume that bidder i's valuation function depends not only on bidder i's private signal, X_i, but also on bidder $-i$'s signal, X_{-i}, which is a random variable to him. This is plausible because if a bidder could know someone else's signal, he would surely revise his valuation, for this would contain valuable information.

If bids are placed alternately between the bidders, let us then denote by a superscript letter the numbering of the bids, keeping in mind that bidder 1's bid will be represented by an odd number, whereas bidder 2's bid will be represented by an even number. If we represent this numbering by k, and if K is the final bid, then the auction will have a series of $k = 1, 2, ..., K$ bids, where K can be an odd or even number, depending on who wins the auction. For example, $E\left[V_i^k\right]$ will denote bidder i's expected valuation of the good at stage k.

Let us make some relevant assumptions.

1. The function v is defined $\forall i$ and $\forall k$ in such a way that $v_i(X_i, X_{-i}, S) = v(X_i, X_{-i}, S)$. This is equivalent to Assumption 1 in Milgrom and Weber, i.e. all the valuations depend on S in the same manner, and each valuation is symmetric in relation to other bidder's signals, at every possible stage in the auction. Moreover, this function is nonnegative, continuous and nondecreasing in its arguments. Neutrality towards risk is implicitly assumed.

2. The valuation function for any bidder i, at any stage k of the auction, has the following property: $E_i\left[V_i^k \mid X_i = x_i, X_{-i} = X_i\right] = x_i$, and this is common knowledge for both bidders. This amounts to assume that when bidder i's estimate of $-i$'s signal, X_{-i}, is equal to his own signal, X_i, his valuation is exactly equal to his private signal, X_i, whose realization is x_i. Obviously, for bidder $-i$ this still holds, but in his case $E_{-i}\left[V_i^k \mid X_{-i} = X_i\right] = X_i$, which is a random variable, whose realization is unknown to him. Moreover, when bidder i is uncertain about whether bidder $-i$ is being rational or irrational with respect to the information made available to him (we use the term "irrational" when bidder i assumes $X_{-i} = X_i$), he will always believe bidder $-i$ is being irrational, hence only taking into account his own private signal, X_{-i}, when taking the expected valuation of the good at any stage. We shall denote this as the Limited Rationality Condition. This assumption will be better understood as soon as we start explaining the model.[1]

3. The reservation price of the seller is zero; every potential buyer or bidder's valuation must fall within the following interval: $0 < E\left[V_i^k\right] < \infty$, for every stage k.

4. There exists a joint density function of the random variables of the model, which is given by $f(x_i, x_{-i}, s)$; the associated distribution function is $F(x_i, x_{-i}, s)$. The variables X_i and X_j, with $i \neq j$, are considered to be conditionally independent, so that their conditional marginal densities $g(.)$ satisfy $g_{X_i,X_{-i}|S}(x_i, x_{-i}|s) = g_{X_i|S}(x_i|s) \cdot g_{X_{-i}|S}(x_{-i}|s)$. The joint distribution of all the random variables will then be $f(x_i, x_{-i}, s) = f_S(s) \cdot g_{X_i|S}(x_i|s) \cdot g_{X_{-i}|S}(x_{-i}|s)$ where $f_S(s)$ is the marginal density of S. There are no uninformative signals, which means that every X_i contains information about the uncertain value of the good, S.

[1] This condition is understandable if we imagine a bidder who always thinks he is outsmarting everyone else, without ever being outsmarted.

After clarifying the basic framework, we can now introduce the auction mechanism and the way through which it works. But first of all it is worth remembering Milgrom and Weber's mechanism for an English auction.

Remark 1 If we denote the valuation function of bidder i by $V_i(X_i, X_{-i}, S)$, then this bidder will bid until the going bid is equal to his maximum expected valuation, which he knows since the beginning. In fact, Milgrom and Weber make use of myopic behaviour to derive this result. They show that this rule of action (or individual strategy) is a best response if every bidder behaves accordingly. In other words, bidder i will follow the optimal strategy $b^*(x_i)$, which tells him to bid (or stay in the auction) until all other bidders quit or the bid reaches $E[V_i | X_i = x_i, X_k = x_k, X_h = X_i]$, where k is the number of bidders who have given up (and whose private signals have already been revealed, by inverting the valuation function), and h the number of bidders still active. This notation has been borrowed from Branco [3]. If this bidder has reached his maximum valuation, then he will give up. If there are bidders continuing, according to this framework, this bidder does not attempt to estimate or guess what these remaining bidder's signals are. For him, these signals will still be random variables, which we have denoted by X_i. In this bidder's case, x_i will be the value assumed by this random variable. In other words, bidder i's best estimate of the other bidder's signals corresponds to his own signal. Milgrom and Weber show that if $b^*(x_i)$ is the optimal strategy for bidder i, then the vector $(b^*(x_i), \ldots, b^*(x_n))$ is an equilibrium point for the English auction with n bidders (see Theorem 10 and its Proof in Milgrom and Weber [5]).

In our setting, we allow bidders to attempt to extract information from the going bids. When bidder $-i$ bids, it is common knowledge that he has a signal, X_{-i}, which is unknown to i. Bidder i will be interested in knowing what value X_{-i} takes. The process through which he tries to guess it is very intuitive: a bid is placed by bidder $-i$ at stage k, given by b^k_{-i}, bidder i observes this bid and thinks bidder $-i$ is acting on his true signal, X_{-i} (which means he thinks bidder $-i$ is "irrational", and he intends to outsmart him - see Assumption 2). For a certain stage k, knowing the previous bid, b^k_{-i}, bidder i also knows that $X_{-i} \geq b^k_{-i}$, because if Assumption 2 is verified (remember that in this case an "irrational" bidder has as his maximum valuation his true signal, X_i) a bid $b^k_{-i} > X_{-i}$ is not optimal. Hence, he thinks that the true signal of bidder $-i$, X_{-i}, has not yet been reached, which means that $X_{-i} \geq b^k_{-i}$ must necessarily be verified (for it is not allowed to place a bid lower than the current bid). Hence, his relevant density function at this stage must take this into account. Thus, every time a bid is submitted, bidder i is allowed to revise his valuation, through the revision of his estimate of bidder $-i$'s signal. For example, if bidder i starts the auction, then his estimate of bidder $-i$'s signal will be given by ${}_i\hat{X}^k_{-i} = E\left[X_{-i}|X_i = x_i\right] = \int_{-\infty}^{+\infty} x_{-i} \cdot f_{X_{-i}}(x_{-i}|X_i = x_i)\,dx_{-i}$, where $f_{X_{-i}}(x_{-i}|X_i = x_i)$ is the marginal density function of $f(x_i; x_{-i}; s)$ with respect to X_{-i} (with x_i known to this bidder), and with $k = 1$ (for it is the first bid).

Then, for bidder i, the expected value of the good at a certain stage (or bid) k can be represented by:

$$
\begin{aligned}
& E\left[V_i^k \middle| X_i = x_i; X_{-i} = h\left(X_i; {}_i\hat{X}^k_{-i}\right)\right] = \\
= \; & E\left[v(X_i; X_{-i}; S) \middle| X_i = x_i; X_{-i} = h\left(X_i; {}_i\hat{X}^k_{-i}\right)\right] \qquad (1)
\end{aligned}
$$

where ${}_i\hat{X}^k_{-i}$ is the expectation by bidder i of bidder $-i$'s signal, at stage k. Some comments are now useful. The main differences between our set-up and Milgrom and Weber's are the estimate by bidder i of bidder $-i$'s signal, X_{-i}: They argue in favour of a sort of myopic behaviour, where the best estimate of bidder $-i$'s signal is given by bidder i's own signal, X_i. This happens because bidder i derives his best reply conditional on his winning the auction. In our model, we argue that this might be possible, but not always the case, because the ascending bid auction provides bidders additional information through the bidding process. We believe that this additional information might be used, and the valuation function of the bidders must take this into account, being this the reason why $X_{-i} = h\left(X_i; {}_i\hat{X}^k_{-i}\right)$. In fact we will argue that $h(.) = \max(.)$, because of the informational content of the signals. If both X_i and X_{-i} are perceived to be of equal quality, then the knowledge of X_{-i} by bidder i should be taken into account in its full extent in i's valuation function. We will come back to this later on, showing that is in fact the optimal form for both bidders: A second point to keep in mind is the fact that this expected valuation corresponds, at stage k, and for bidder i, to his maximum valuation of the good, because it is making use of all the information available. He should have no incentives whatsoever to place a bid higher than this value, for it would leave him with a negative payoff if he won the auction.

Hence, the bidding behaviour in the auction should be described by a bidding function which converts the optimal strategy (or maximum valuation) into a bid, at every stage. This bidding function, for bidder i, must be a function of his maximum valuation at that moment. Let us denote this bidding function by $b_i^k = b\left(E\left[V_i^k \middle| X_i = x_i; X_{-i} = h\left(X_i; {}_i\hat{X}^k_{-i}\right)\right]\right)$. This function must have some properties, so that it does not violate the rules of the auction. Hence, its values (or bids) must lie within the interval given by:

$$\begin{aligned} & b^{k-1}_{-i}\left(E\left[V^k_{-i} \mid X_{-i}=x_{-i}; X_i = h\left(X_{-i}; \hat{X}^{k-1}_{i}\right)\right]\right) < \\ & < b^k_i\left(E\left[V^k_i \mid X_i = x_i; X_{-i} = h\left(X_i; \hat{X}^k_{-i}\right)\right]\right) \le \\ & \le E\left[V^k_i \mid X_i = x_i; X_{-i} = h\left(X_i; \hat{X}^k_{-i}\right)\right] \quad ; \text{ for } i = 1; 2 \end{aligned} \tag{2}$$

This bidding function transmits two ideas: firstly, the ascendingness of the bids. The current bid to be placed by bidder i must be strictly higher than the previous bid, at stage $k-1$, by bidder $-i$; given by b^{k-1}_{-i} (notice that only this bid is revealed to bidder i; he then has to infer or guess what the maximum valuation behind it is). Secondly, it is bounded above by the expected valuation of bidder i at stage k. Again, he should not bid more than his maximum valuation, because this would leave him with a negative payoff. It is worth noting the difference between this upper limit (or reservation valuation) and Milgrom and Weber's. Our upper limit is allowed to vary as the auction is taking place, because bidder i is constantly processing the information being disclosed through the bids, and hence updating his estimate of bidder $-i$'s signal. Let us now show the way in which this might happen.

At a certain stage k, such that $1 < k < K$; bidder i will have to decide whether to continue in the auction or not. At this point he knows what were the previous bids in the auction, i.e. he knows $\left[b^1_i; b^2_{-i}; \ldots; b^{k-1}_{-i}\right)$ (notice that $i = 1$ in this case). If this bidder is rational, he will attempt to extract information from all the previous bids, especially the last bid by bidder $-i$. His initial private signal is unchangeable, but he can attempt to estimate bidder $-i$'s initial signal, which is unknown to him. In order to do this, he will have to consider the updated density function which, at this stage, takes the following form:

$$f\left[x_{-i}; s \mid X_i = x_i; X_{-i} \ge b^{k-1}_{-i}\right) = \begin{cases} \dfrac{f(x_i; x_{-i}; s)}{\int_{b^{k-1}_{-i}}^{+\infty} f_{X_i; X_{-i}}(x_i; x_{-i})dx_{-i}}; & X_{-i} \ge b^{k-1}_{-i} \\ 0; & \text{otherwise} \end{cases} \tag{3}$$

This should be the relevant density function because bidder i thinks bidder $-i$'s expected valuation is higher than his last bid, b^{k-1}_{-i}: So he knows that $X_{-i} \ge b^{k-1}_{-i}$: Further, he knows that according to the rules the bids must be increasing, so he is ruling out the points corresponding to all the previous bids, i.e. points which are lower than b_{-i}^{k-1}. Notice the importance of Assumption 2 in this context: bidder i knows that $E\left[V_{-i}^{k-1} \mid X_i\right] = \left[X_{-i}\right] = \left(X_{-i}\right)$, which means that instead of having $f\left(x_{-i}; s \mid X_i = x_i; E\left[V_{-i}^{k-1}\right] \geq b_{-i}^{k-1}\right)$ in (3), we can rearrange the second term in the conditioning to yield $X_{-i} \geq b_{-i}^{k-1}$. This is also true because if bidder i is not sure about whether bidder $-i$ is rational, he will assume he is irrational. If this is the case, then an irrational bidder will only consider his own private signal when taking the expected valuation of the good, and the substitution just mentioned is allowed. Even though this is not a necessary assumption, it is a very simplifying one, for it makes changes in variables unnecessary, especially when considering probabilities.

We can then define the estimate of bidder $-i$'s signal by bidder i at stage k; ${}_{i}\hat{X}_{-i}^{k}$; to be:

$$
\begin{aligned}
{}_{i}\hat{X}_{-i}^{k}\left(b_{-i}^{k-1}\right) &= E\left[X_{-i} \mid X_{-i} \geq b_{-i}^{k-1}; X_i = x_i\right] \\
&= \int_{b_{-i}^{k-1}}^{+\infty} x_{-i} . f_{X_{-i}}\left(x_{-i} \mid X_{-i} \geq b_{-i}^{k-1}; X_i = x_i\right) dx_{-i} \\
&= \frac{\int_{b_{-i}^{k-1}}^{+\infty} x_{-i} . f_{X_i; X_{-i}}\left(x_{-i}\right) dx_{-i}}{\int_{b_{-i}^{k-1}}^{+\infty} f_{X_i; X_{-i}}\left(x_{-i}\right) dx_{-i}}
\end{aligned} \tag{4}
$$

The term $f_{X_{-i}}\left(x_{-i} \mid X_{-i} \geq b_{-i}^{k-1}; X_i = x_i\right)$ is obtained from (3) by integrating out with respect to s. With this new information at this stage k; bidder i may revise his maximum valuation and hence his strategy:

$$
b_i^{*k}\left(x_i \mid b_{-i}^{k-1}\right) = E\left[V_i^k \mid X_i = x_i; X_{-i} = \max\left(X_i; {}_{i}\hat{X}_{-i}^{k}\right)\right] \tag{5}
$$

This strategy is a rule telling bidder i to bid at stage k if and only if his bid (given by the bidding function, which converts this maximum valuation into a bid) is not higher than his expected valuation at that stage. The main difference between our bidding behaviour and Milgrom and Weber's is related to bidder $-i$'s signal. They consider bidders to behave myopically, which means that at every stage in the auction they don't attempt to estimate what bidder $-i$'s true signal is. They consider it to be the same as theirs until their maximum valuation is reached, moment after which they quit the auction. Our bidding behaviour argues in favour of rational agents, who try to take advantage of the information being revealed through the bids at every stage in the auction. In this case, they have an estimate for the other bidder's signal (which may obviously be wrong), and they include it in their valuation function, if and only if this estimate is higher their own private signal, X_i. Remember that the signals are considered to be of equal quality. The reasons behind the use of $X_{-i} = \max\left(X_i, \hat{X}^k_{-i}\right)$ are related to the quality of the signals. Because they have the same quality, the best estimate of X_{-i} should contain all the information available at that moment (at stage k, in this case). If all the information available leads to a value of $\hat{X}^k_{-i} < X_i$, then bidder i may think that his own signal, X_i, is still the best estimate of X_{-i}. However, if at stage k, and in possession of all the information disclosed through bids until this stage, $\hat{X}^k_{-i} > X_i$, then bidder i has reasons to believe that bidder $-i$'s signal, X_{-i}, is in fact higher than X_i, and the consequence is to substitute X_i for $\hat{X}^k_{-i}$ as the best estimator of X_{-i}. There could be several functional forms of $h(\cdot)$ to perform this substitution. We have chosen the maximum rule, i.e. $X_{-i} = h\left(X_i, \hat{X}^k_{-i}\right) = \max\left(X_i, \hat{X}^k_{-i}\right)$, because the signals are considered to be of equal quality, which makes it indifferent for bidder i to choose between X_i and $\hat{X}^k_{-i}$ when they are the same. There are no reasons whatsoever for bidder i to believe that $X_{-i} = X_i$ when a stage of the auction has been reached (and information has been disclosed) such that $\hat{X}^k_{-i} > X_i$. The verification of this inequality is a signal for bidder i that there is information available at that stage that is useful for him to improve his information about the uncertain value of the good, and hence his own valuation. If this is so, it does not make sense to consider X_i as a good estimator of X_{-i}, because it is henceforth informationally inferior.

Notice that (5) does not include information about bidder $-i$ at the following stage $k+1$, which means that bidder i prefers not to consider bidder $-i$'s reaction to his bid. Hence, any bidder does not infinitely (or finitely until stage K) anticipate the other bidder's reaction to his bid, because there is no advantage in doing it. It is obvious that if he did, then bidder $-i$'s reply at stage $k+1$ would try to anticipate bidder i's move at stage $k+2$, and so on until the end of the auction. In other words, moving backwards from the last stage K, bidder i, at stage k, will choose to ignore information which is only revealed in stages $k+1, \ldots, K$. Let us then understand why such situation is ruled out. If, at stage k, bidder i considered bidder $-i$'s response at $k+1$, his optimal strategy (or maximum valuation) would be:

$$b_i^{*k}\left(x_i \mid b_{-i}^{k-1}\right) = E\left[V_i^k \,\middle|\, \begin{array}{l} X_i = x_i,\ X_{-i} = \max\left(X_i, \hat{X}^k_{-i}\right), \\ b_{-i}^{*k+1}\left(x_{-i} \mid b_i^k\right) = \\ \quad E\left[V_{-i}^{k+1} \,\middle|\, \begin{array}{l} X_{-i} = x_{-i}, \\ X_i = \max\left(X_{-i}, \hat{X}_i^k\right), \\ b_i^{*k+2}\left(x_i \mid b_{-i}^{k+1}\right) = \cdots \end{array} \right] \end{array} \right] \quad (6)$$

Keeping Assumption 2 in mind (any bidder believes that the other is irrational if he has no more information), we can see that the anticipation of bidder $-i$'s valuation at stage $k+1$ by bidder i does not reveal any new information:

$$
\begin{aligned}
& E_i\left[E_{-i}\left[V_{-i}^{k+1} \,\middle|\, X_{-i}=x_{-i}, X_i=\max\left(X_{-i}, -\hat{X}_i^k\right), b_i^{k+2}\left(x_i \mid b_{-i}^{k+1}\right)\right]\right] = \ldots \\
& = E_i\left[V_{-i}^{k+1} \,\middle|\, X_{-i}=x_{-i}, X_i=X_{-i}\right] = E\left[X_{-i}\right] \qquad (7)
\end{aligned}
$$

Thus, trying to anticipate bidder $-i$'s reply will bring him back to the same initial problem: the fact that he does not know bidder $-i$'s true signal, x_{-i}. If this is the case, then there is no informational gain in attempting to anticipate the next moves by the other bidder, and (5) should in fact be the correct bidding strategy. We can then generalize this behaviour for a general stage k, for both bidders, into the following proposition:

Proposition 1 *For a stage k that satisfies $E\left[V_i^k \mid X_i = x_i, X_{-i} = \max\left(X_i, -\hat{X}^k_{-i}\right)\right] \geq b_i^k(\cdot)$, with $-\hat{X}^k_{-i} > X_i$, where $i = 1, 2$, the bidding strategy (or maximum valuation) at this stage k is given by $b_i^{*k}\left[x_i \mid b^{k-1}_{-i}\right] = E\left[V_i^k \mid X_i = x_i, X_{-i} = -\hat{X}^k_{-i}\right]$, taking into account the additional information disclosed by the previous bid. This additional information is perceived and compiled by bidder i into the term $-\hat{X}^k_{-i}\left(b^{k-1}_{-i}\right) = E\left[X_{-i} \mid X_{-i} \geq b^{k-1}_{-i}, X_i = x_i\right]$, given by (4), which gives bidder i's estimate of bidder $-i$'s signal at stage k. Thus, bidder i will bid if the resulting bid is not higher than his maximum valuation at this stage k. For a stage where $-\hat{X}^k_{-i} > X_i$ is not verified, the bidding behaviour is given by Milgrom and Weber's strategy, $b^*(x_i) = E\left[V_i^k \mid X_i = x_i, X_{-i} = X_i\right]$.*

Proof. An alternative to using (5) as the bidding behaviour could be the use of Milgrom and Weber's. Under their framework, the optimal bidding strategy, $b^*(x_i)$, does not take into account the information which arrives through the bids (see Remark 1). We will denote this course of action as "irrational". We will then contrast this rule to ours, given in (5), which we shall denote by "rational", and which takes into account the term $-\hat{X}^k_{-i}$. Keeping in mind that an increase in a bidder's valuation must be caused by additional (positive) information about the quality of the good (confirm this in (1)), we shall show that the optimal strategy for each bidder is taken from a game repeated K times (the number of stages in the auction), where each bidder alternatively chooses the best rule to follow (either "rational" or "irrational"). There are many possible combinations of rules to follow during the K stages of the auction, but by iterated strict dominance (or rationalizability of strategies), we can reduce this set to a smaller number of rational actions, because many are strictly dominated. Milgrom and Weber show that the "irrational" strategy is optimal, where this strategy is given by playing the "irrational" rule during stages $1, 2, \ldots, K$. In other words, $b_i^*(x_i)$ is an optimal strategy, and, because of symmetry, the vector $(b^*(x_i), \ldots, b^*(x_n))$ is an equilibrium point of the English auction (see Proof in Milgrom and Weber [5]). We believe that our "rational" strategy is also optimal, where we define "rational" to be a strategy that uses the rule "irrational" (Milgrom and Weber's) until $\hat{X}^k_{-i} > X_i$ is verified, and which changes to "rational" afterwards [2]

Let us then show why this is so. We can then see what the outcomes of this game are, and also see which strategies are likely to form an equilibrium. The game to be played from stage $k = 1, \ldots, K$, and its payoffs (notice that we did not display the current bid, which should be subtracted from the maximum valuation), is given by:

	$-i$ plays "rational"	$-i$ plays "irrational"
i plays "rational"	$\left(E\left[V_i^k\left(X_i, \hat{X}^k_{-i}, S\right)\right]; E\left[V^k_{-i}\left(X_{-i}, \hat{X}^k_{i}, S\right)\right]\right)$	$E\left[V_i^k\left(X_i, \hat{X}^k_{-i}, S\right)\right]$; $E[V_{-i}(X_{-i}; X_{-i}; S)]$
i plays "irrational"	$\left(E[V_i(X_i; X_i; S)]; E\left[V^k_{-i}\left(X_{-i}, \hat{X}^k_{i}, S\right)\right]\right)$	$E[V_i(X_i; X_i; S)]$; $E[V_{-i}(X_{-i}; X_{-i}; S)]$

Table 1: The game to be played by each bidder every time it is his time to bid

It can clearly be seen that at each stage the optimal strategy will be for bidder i (and because of symmetry also to bidder $-i$) to play the "rational" rule, as long as $\hat{X}^k_{-i} > X_i$, and play "irrational" otherwise. Not only does the latter dominate Milgrom and Weber's rule after $\hat{X}^k_{-i} > X_i$ is verified but it also corresponds to a dominant equilibrium. This has quite an intuitive explanation. If bidder i thinks his signal has more information than bidder $-i$'s signal estimate, he will only consider his own signal when valuing the good. This is described by the "irrational" rule, which is the most likely at earlier stages in the auction. But as the bids come closer to X_i, the probability of bidder $-i$'s signal, X_{-i} being higher than bidder i's is increasing, the same happening with the expected value of $-i$'s signal, $\hat{X}^k_{-i}$. When $\hat{X}^k_{-i}$ becomes bigger than X_i, this bidder will operate a change in his rule of behaviour, and start playing "rational". Again, this is explained because bidder i thinks that bidder $-i$'s signal might contain more information about the uncertain

[2] Notice that it does not change back to "irrational" under any circumstances. This happens because $\hat{X}^k_{-i}$ is increasing in the bids, as long as bidder $-i$ is active.

quality than his own signal, so he decides to start paying attention to ${}_i\hat{X}^{k}_{-i}$ when valuing the good. Needless to say, from this point onwards the quitting point of bidder i will no longer be his signal, but a point higher than x_i[3].

To determine the equilibrium strategy of this game, let us think of the optimal action for bidder i at stage K, the last stage of the auction. Remember that he always believes bidder $-i$ to be playing "irrational" (Assumption 2). At the final stage, there are several possibilities:

1. Bidder i was playing "rational" and he has reached his maximum valuation, $E\left[V_i^{K-1}\left(X_i, {}_i\hat{X}^{K-1}_{-i}; S\right)\right]$. His expected payoff is 0 (because he places a final bid equal to his valuation), bidder $-i$ wins the auction and pays a price $p^* = b_i^{K-1}\left(E\left[V_i^{K-1} \mid X_i = x_i; X_{-i} = {}_i\hat{X}^{K-1}_{-i}\right]\right) = E\left[V_i^{K-1} \mid X_i = x_i; X_{-i} = {}_i\hat{X}^{K-1}_{-i}\right]$.

2. Bidder i was playing "rational" and bidder $-i$ quits before his maximum valuation is reached. In this case, bidder i gets the good for a price $p^* = b^{K-1}_{-i}$ (notice that if bidder $-i$ was playing "irrational" all the time, adverse selection occurs, for the bidder with the highest initial signal ends up not getting the good!). This is understandable if bidder i has an optimistic estimate of bidder $-i$'s signal, in which case he trusts in that signal so much that he is willing to follow it until the end ("forcing" bidder $-i$ to quit). His expected payoff will be $E\left[V_i^{K} \mid X_i = x_i; X_{-i} = {}_i\hat{X}^{K}_{-i}\right] - b^{K}_{-i}$, which is larger than 0 if $v\left(X_i, {}_i\hat{X}^{K}_{-i}\right) > v\left(X_{-i}, X_{-i}\right)$[4].

3. Bidder i was playing "irrational", and he reached the point of his maximum valuation, so that $E\left[V_i^{K-1} \mid X_i = x_i; X_{-i} = X_i\right] - b_i^{K-1}(.) = 0$. If ${}_i\hat{X}^{k}_{-i} > X_i$ is not verified, bidder i has no incentive to switch to "rational", and he quits at this point. His final payoff is 0. The final price to be paid by bidder $-i$ is $p^* = b^*(x_i)$.

4. Bidder i was playing "irrational" and bidder $-i$ quits. Bidder i wins the auction paying a price $p^* = b^*(x_{-i})$ and his expected payoff will be

[3] Remember that $E\left[V_i^{k} \mid X_i = x_i; X_{-i} = X_i\right] = x_i$, for bidder i. Because the valuation function is nondecreasing in its arguments, it follows naturally that $E\left[V_i^{k} \mid X_i = x_i; X_{-i} = {}_i\hat{X}^{k}_{-i}\right] > x_i$, if ${}_i\hat{X}^{k}_{-i} > X_i$.

[4] Notice that $x_i < x_{-i}$, which means that for this condition to hold, ${}_i\hat{X}^{K}_{-i}$ must be significantly higher than x_{-i}, to compensate. This is what we mean by "overestimation" of his opponent's signal.

$E\left[V_i^K \mid X_i = x_i\right] - b^*(x_{-i})$, which is positive if $x_i > x_{-i}$ (see Proof of Theorem 10 in Milgrom and Weber [5]).

From these four possibilities, we can now show that as long as $X_i > {}_i\hat{X}^k_{-i}$, "irrational" is the best rule to choose, i.e. Milgrom and Weber's equilibrium strategy is valid. And this is so because including the term ${}_i\hat{X}^k_{-i}$ in bidder i's valuation function (replacing X_i as the best estimate of bidder $-i$'s signal) has no effects at all. When a stage k is reached such that ${}_i\hat{X}^k_{-i} > U_i$, this bidder now has a new equilibrium rule, telling him to replace his "irrational" estimate of bidder $-i$'s signal for his rational estimate, given by ${}_i\hat{X}^k_{-i}$. From this point onwards, he finds it impossible that bidder $-i$'s true signal is lower than his (remember that $E_i\left[V_i^k \mid X_i = x_i; X_{-i} = X_i\right] = x_i$ and $E_{-i}\left[V_i^k \mid X_{-i} = X_i\right] = X_i$ - see Assumption 2), and he knows what might happen at the final stage K. Under no circumstances will he be worse off than by giving up at this stage $k < K$, because the worse that could happen is him ending up with a 0 payoff (which he would end up getting had he given up at this stage k). In fact, he will face a higher probability of winning if he plays "rational", without incurring in negative payoffs, because of his bidding above the point where his true signal has been reached. He also knows he might drive bidder $-i$ out of the auction, by extracting all the information from him. Thus, equation (5) should represent the bidding strategy (or maximum valuation) of this bidder for values of ${}_i\hat{X}^k_{-i} > X_i$, telling him to bid if the corresponding bid is not higher than this updated maximum valuation.

If we denote the "irrational" rule at stage k with I_k (corresponding to Milgrom and Weber's $b^*(x_i)$), and the rational with R_k (corresponding to our strategy, $b_i^{*k}\left(x_i \mid b^{k-1}_{-i}\right)$), the equilibrium strategy of this game for bidder i, which we denote by b_i^*, is given by $b_i^* = (I_1, I_3, ..., R_k, R_{k+2}, ..., R_K)$, for $i = 1$ (and assuming this is the winning bidder, so that his last bid is at the final stage K) if there exists a stage k such that ${}_i\hat{X}^k_{-i} > X_i$, and bidder $-i$'s optimal strategy will be $b^*_{-i} = (I_2, I_4, ..., R_j, ..., R_{K-1})$; once again if a stage j exists such that ${}_{-i}\hat{X}^j_i > X_{-i}$. ■

It is worth noting that in this case, the winner of the auction may or may not incur in negative profits. The reason why he stays in the bidding process until this point is that in expectation he gets a positive payoff. However, when the auction finishes and all the information is revealed, he may realize that his payoff was negative, and he will have incurred in the winner's curse. This may explain why 30 to 40% of the auctions in Avery and Kagel[1] yielded negative payoffs for the winners. In their study, however, it is not possible to see whether the high signal bidder wins the auction with probability 1. Our framework suggests the this probability may be high but lower than 1.

We can now relax the Limited Rationality Condition, and get the result that under this framework bidders will bid at least as high as in a second-price auction (Milgrom and Weber's result is the lower bound of our results) and at most they will bid according to our Proposition 1.

Proposition 2 *In this framework, without the Limited Rationality Condition, bidding up to less than* $b_i^{*k}\left(x_i | b_{-i}^{k-1}\right) = E\left[V_i^k \middle| X_i = x_i; X_{-i} = X_i\right]$ *or more than* $b_i^{*k}\left(x_i | b_{-i}^{k-1}\right) = E\left[V_i^k \middle| X_i = x_i; X_{-i} = \hat{X}_{-i}^k\right]$, *for* $i = 1, 2$ *is not optimal. These will be the lower and upper bounds of our results.*

Proof. Notice that no bidder has an incentive to bid less than $b_i^{*k}\left(x_i | b_{-i}^{k-1}\right) = E\left[V_i^k \middle| X_i = x_i; X_{-i} = X_i\right]$, for $i = 1, 2$ because this is the symmetric equilibrium. In other words, this is the best reply for i when $-i$ adopts this strategy. On the other hand, with the Limited Rationality Condition, we get the highest possible price, because bidders are trying to outsmart each other, and it is only when the auction finishes that they realize that.

When bidder i takes into account the possibility that bidder $-i$ is trying to outsmart him by inverting his bid, he will be more cautious, and almost surely take this into account. In other words, he will realize that he may incur in the winner's curse if he wins, and he bids more conservatively. The exact end point depends on each bidder's beliefs, but it may lie anywhere in between the interval defined in the Proposition. ■

This result is quite sensible. If a bidder feels someone is trying to outsmart him by extracting information from his bid, and if he is doing the same, then he knows he is providing some misinformation which may damage him. This damage will be the winner's curse if he wins. So basically he will be more careful when trying to outsmart the other bidder, and surely bid less than under the Limited Rationality Condition. When both bidders behave in this way, not only will there be an equilibrium point in the auction (Proposition 1) but it will also lie in the interval defined in Proposition 2.

3 The Auction with Herd Behaviour

We will now attempt to show that as soon the "rational" rule starts being played, some sort of herd behaviour might arise. Intuitively, this amounts to realize that when a player includes his expectations in his valuation function, then nothing prevents him from taking this even further. Let us imagine a stage z; with bidder i playing "rational", in which case:

$$E\left[V_i^z | X_i = x_i; X_{-i} = \hat{X}_{-i}^z\right] - b_i^z(\cdot) = 0 \tag{8}$$

At this stage, bidder i has reached his maximum valuation of the good, meaning that the bid he would place is equal to that valuation. In other words, this bidder is indifferent between winning and losing at this stage. Let us assume he bids. What will happen in stage $z+2$ (assuming that bidder $-i$ does not stop bidding at stage $z+1$)? From the notion of equilibrium, this bidder should quit, because he has no incentive whatsoever in continuing. If i continues in the bidding process, he will certainly have a negative payoff, which is obviously not optimal and hence, he should stop bidding.

To understand why, during the auction, and after switching to the "rational" rule previously described, a bidder might temporarily and partially ignore his signal, let us assume that the density function given by $f(x_i, x_{-i}, s)$ is discrete. We can then make use of the following definitions (for bidder i):

Definition 1 At a certain stage k of the auction, let $\Pr[S = x_i | X_i = x_i] = p$ denote the probability of bidder i's signal being correct (after receiving the signal), and $\Pr\left[S = -\hat{X}^k_{-i} \,\middle|\, X_{-i} = -\hat{X}^k_{-i}\right] = q$ denote the probability of bidder i's estimate of bidder $-i$'s signal being the correct estimate of the uncertain S.

An application of Bayes's rule tells us that:

$$\Pr[S = x_i | X_i = x_i] = \frac{\Pr[X_i = x_i | S = x_i] \cdot \Pr[S = x_i]}{\Pr[X_i = x_i]} = p \tag{9}$$

in a way similar to Scharfstein and Stein [6] and the same application can be used for q. This procedure can gives us the a priori probabilities (before receiving the signal) of the signal being the correct one. Making use of the joint density, we can rewrite (9) as:

$$\Pr[S = x_i | X_i = x_i] = \frac{f_{X_i,S}(x_i, x_i)}{f_{X_i}(x_i)} = \frac{\Pr[X_i = x_i | S = x_i] \cdot f_S(x_i)}{f_{X_i}(x_i)} = p \tag{10}$$

which means that p computable. Now note that the following three events are mutually exclusive: either x_i, $-\hat{X}^k_{-i}$ or some scalar $\lambda \neq x_i \neq -\hat{X}^k_{-i}$, with $\lambda, -\hat{X}^k_{-i} \in [\underline{s}, \overline{s}]$ (the domain of S) is the correct estimate of S. If this is so, then the following equation must hold for any stage k in the auction:

$$\begin{aligned}
1 = \Pr & \left[(S = x_i | X_i = x_i) \cap \left(S \neq -\hat{X}^k_{-i} \,\middle|\, X_{-i} = -\hat{X}^k_{-i} \right) \right] + \\
+ \Pr & \left[\left(S = -\hat{X}^k_{-i} \,\middle|\, X_{-i} = -\hat{X}^k_{-i} \right) \cap (S \neq x_i | X_i = x_i) \right] + \\
+ \Pr & \left[\left(S \neq -\hat{X}^k_{-i} \,\middle|\, X_{-i} = -\hat{X}^k_{-i} \right) \cap (S \neq x_i | X_i = x_i) \right]
\end{aligned} \tag{11}$$

Using the notation defined above, we can rewrite (11) as:

$$p(1-q) + q(1-p) + (1 - p(1-q) - q(1-p)) = 1 \tag{12}$$

As the auction is taking place, p does not change, for it is a probability which has been fixed initially with the arrival of the signal x_i to bidder i. However, q is a probability constantly changing during the auction, because $_{-i}\hat{X}^k_{-i}$ is an estimate being revised every time a new bid is submitted. Thus, during the auction, bidder i has an idea about the accuracy of bidder $-i$'s signal, and he should use it. It is then quite natural to assume that if:

$$p(1-q) > q(1-p) + (1 - p(1-q) - q(1-p)) \tag{13}$$

bidder i has reasons to believe that his signal is the most accurate, for values of q and p that verify equation (12). Likewise, if:

$$q(1-p) > p(1-q) + (1 - p(1-q) - q(1-p)) \tag{14}$$

at a certain stage k, then bidder i has reasons to believe that the other bidder's signal is more accurate than his own, given all the information available at that stage. But if the latter is true, why should bidder i believe in his signal? If $_{-i}\hat{X}^k_{-i}$ is more likely to be the true estimate of S, why should bidder i link his valuation to a less informative signal? We shall refer to equation (14) as a Strong Condition for herding.

However, a less strict condition must be met so that herding occurs. For probabilities p and q that satisfy (12), the following condition must also be satisfied:

$$p(1-q) \cdot x_i + q(1-p) \cdot {}_{-i}\hat{X}^k_{-i} > x_i \tag{15}$$

This condition, which we shall call Weak Condition for herding, tells us that bidder i will only herd if for the probabilities given above, i.e. the weighted average (with weights equal to the probability of each signal being correct) of his signal, x_i, and of his estimate of bidder $-i$'s signal, $_{-i}\hat{X}^k_{-i}$, is higher than his initial private signal. The interpretation of this fact is straightforward: bidder i will only abandon partially his signal if there is some additional information to take advantage of. When (15) is satisfied, bidder i realizes that the term $_{-i}\hat{X}^k_{-i}$ weighted by $q(1-p)$; the probability of this being the correct uncertain value of the good, together with his own signal (and the probability $p(1-q)$) are a better estimator of S than x_i alone. Let us then elaborate the following proposition:

Proposition 3 If, at any stage k in the auction, with bidder i playing "rational", $q(1-p) > p(1-q) + (1 - p(1-q) - q(1-p))$ holds (i.e. the Strong

Condition), with values of p and q that satisfy (12), then bidder i should partially or totally ignore his own private signal, x_i. The way in which he should do this is to attach a weight both to his signal and to $\hat{X}^k_{-i}$ when valuing the good. Hence, his expected valuation of the good at this point should be given by $E\left[V_i^k \mid X_i = p(1-q)\cdot x_i + q(1-p)\cdot \hat{X}^k_{-i}; X_{-i} = \hat{X}^k_{-i}\right]$ if $p(1-q)\cdot x_i + q(1-p)\cdot \hat{X}^k_{-i} > x_i$, i.e. when the Weak Condition is satisfied, and by $E\left[V_i^k \mid X_i = \hat{X}^k_{-i}; X_{-i} = \hat{X}^k_{-i}\right]$ if the latter is not satisfied, which makes bidder i totally abandon his private signal, x_i. In any of the cases, this will be his new maximum valuation (note that it is higher than $E\left[V_i^k \mid X_i = x_i; X_{-i} = \hat{X}^k_{-i}\right]$ because bidder i is playing "rational", which means that $\hat{X}^k_{-i} > x_i$; and the valuation function is non decreasing in its arguments). This is the result when the Strong Condition for herding, equation (14), holds.

If only the Weak Condition, equation (15), is met, then his expected valuation will be given only by $E\left[V_i^k \mid X_i = p(1-q)\cdot x_i + q(1-p)\cdot \hat{X}^k_{-i}; X_{-i} = \hat{X}^k_{-i}\right]$, and the bidder will not abandon his initial signal completely.

Proof. The proof of this proposition follows closely the proof of Proposition 1, so we should always have in mind the game which is played every turn. Some steps will not be presented.

If (14) holds for values of p and q that satisfy (12), then bidder i perceives bidder $-i$'s signal to be more accurate than his own. He basically faces two options: either he links his valuation to $\hat{X}^k_{-i}$ only, or he weighs each piece of information with the respective probabilities. The former is a radical solution, from the point of view of bidder i, which brings about herd behaviour in its full extent. The latter, however, is a less radical possibility, which should also be considered.

When bidder i perceives his signal to be less informative than $\hat{X}^k_{-i}$, he can adjust his valuation function to this fact, by attaching a weight corresponding to the respective probabilities to each of those events. Hence, his private valuation, or signal, should incorporate this new information, and his private signal X_i will now be given by $X_i = p(1-q)\cdot x_i + q(1-p)\cdot \hat{X}^k_{-i}$. However, this procedure should only be used if $p(1-q)\cdot x_i + q(1-p)\cdot \hat{X}^k_{-i} > x_i$. It is quite straightforward to see why: if this last condition (the Weak Condition) is not satisfied and equation (14) is (the Strong Condition), bidder i will be wasting information. This waste is visible, because knowing that $\hat{X}^k_{-i}$ is the more likely value of S, this bidder will be linking his valuation function to an even lower value estimate of S, given by $p(1-q)\cdot x_i + q(1-p)\cdot \hat{X}^k_{-i}$. By using backward induction, if he loses the auction, it will be because his maximum expected valuation of the good was too low, as he ignored the fact that $\hat{X}^k_{-i}$ was the most likely estimate. If he is rational, he will not want this outcome when the Weak Condition is not verified and equation (14), the Strong Condition, is. For the waste of information to be minimized when $p(1-q) \cdot x_i + q(1-p) \cdot \hat{X}^k_{-i} < x_i$ holds (when the Weak Condition fails to hold), this bidder should link his valuation to $\hat{X}^k_{-i}$ only, to take advantage of the fact that this is the most likely value to be assumed by S. Hence, his maximum valuation should be given by $E\left[V_i^k \mid X_i = \hat{X}^k_{-i}; X_{-i} = \hat{X}^k_{-i}\right]$ when this is the case.

If only the Weak Condition holds, then herding will occur but not in its full extent. The initial signal will not be completely abandoned. If both Conditions are verified, the same result is valid, which means that the initial signal is never totally abandoned. Notice this is an attempt to minimize the possibility of occurrence of herd behaviour. Under different assumptions, herding could be more likely, but hardly less likely. Notice that the Weak Condition is a necessary and sufficient condition for "partial" herd behaviour to occur, whereas the Strong Condition is only a necessary condition for "total" herd behaviour to occur. ■

4 Conclusion

Milgrom and Weber [5] describe the general English auction setting, and show what the outcome of such auction should be at the symmetric Nash equilibrium. We have used their model as the starting point and have shown that as the ascending auction goes on, bidders could take into account the information being disclosed through bids. Even though bids are just numbers, they reflect the information set and the optimal strategy of the bidder who has placed them and this may have as a behavioural consequence the existence of the Bayesian updating process which we have described. A rational bidder should take the current bid into account, and revise his expected valuation of the good every time a bid is submitted. The end point of such a rational bidder may be higher than the end point of Milgrom and Weber's strategy, as shown in our Proposition 1. Hence, the final prices in our model could be higher than under their framework.

A very interesting result is derived, which makes the auction inefficient. Under the conditions described by the model, it is possible that the bidder with the higher initial signal about the uncertain value of the good ends up not getting it, i.e. he loses the auction. Again this can be described by the existence of the Bayesian updating. The higher the estimate of his opponent's signal, the longer a particular bidder will stay active. In fact, when that estimate is clearly overestimated, he may stay active so long as to drive the high signal bidder out of the auction, and realize negative payoffs.

An additional complication is introduced with the possible occurrence of herd behaviour, which will make a bidder ignore, totally or partially, his own private signal about the uncertain quality of the good for sale. If it does occur, then the final price might be even higher, and this might help to explain the extremely high prices in some ascending bid auctions, for example art auctions.

We have used the simple case of two bidders. The extension to the case of n bidders will probably exacerbate the possibility of herd behaviour and increase the possibility of rational bidders being present in the auction, driving the prices to even higher values. Another very interesting extension would be to analyse the case of completely rational bidders, who would always assume to be facing agents similar to them in the auction. In a way, our Proposition 2 gives the lower and upper bounds of bidding in this case, but does not attempt to show under which conditions we would be closer to one or another. However, our conjecture is that the more rational bidders there are, the more likely it is that the end point is the initial symmetric equilibrium, as defined by Milgrom and Weber, and hence the lower the revenue for the seller as compared to the results of our paper. This could be so because the more rational each bidder is, the more he knows that his opponent is trying to infer something from his bid, and acting upon it. So he knows that if he does the same, he runs the risk of winning the auction and incurring in a negative payoff (winner's curse). So he will be reluctant in bidding much above the symmetric equilibrium. If his opponent thinks in the same way, the symmetric equilibrium will be the more likely end point.

Another remark is that under the conditions described by our model, the Revenue Equivalence Theorem first suggested by Vickrey [7] does not hold, because the expected price of the English auction is higher than the expected price of the second-price sealed bid auction, the conclusion also reached by Milgrom and Weber [5].

A final point is worth mentioning. Our framework may help to explain real world and experimental evidence that in a significant proportion of auctions, the winner realizes negative payoffs. Unfortunately, in Avery and Kagel's [1] study it is not possible to know whether the high signal bidder won the auction almost always in the standard auction.

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